

# A New Ionospheric Model for Wide Area Differential GPS: The Multiple Shell Approach

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## ABSTRACT

Global Ionospheric Modeling (GIM) algorithms have been recently enhanced to solve for electron content distributions on multiple horizontal grids distributed vertically (multiple shell), instead of using a single grid at a fixed height (single shell). We are assessing this new ionospheric model for application in Wide Area Differential GPS (WADGPS) systems over the coterminous United States (CONUS). The additional parameters from multiple vertical layers allow GIM to better model the height variation of ionospheric electron density along the GPS raypaths, and accommodate significant diurnal height variations of the ionosphere which are ignored in a fixed-height single layer approach. This new model is a conceptually a simple extension of several existing WADGPS algorithms, that may offer benefits similar to various forms of ionospheric tomography. We compared solutions that model the ionosphere as a correlated random-walk stochastic process (the standard GIM approach), with an older strategy assuming the ionospheric centroid height to be at a fixed height. It is shown that the multi-shell approach improves slant ionospheric delay accuracy and reduces systematic error in the GPS inter-frequency bias estimates.

## INTRODUCTION

One of the major error sources in GPS positioning is ionospheric refraction which causes signal propagation delays. The disturbing influences of the temporally and spatially varying ionization of the ionosphere have great impact on satellite navigation using GPS. Dual-frequency observations can be used to eliminate almost all of the ionosphere's effect. To correct data from a single-frequency GPS receiver for the ionospheric effect, there are several techniques that one can use.

We can ignore the effect but then we have to live with the consequences. They can be quite severe given the fact the measurement error caused by the ionosphere can be as

significant as about 50 meters on L1. Multiplying this number by a typical Position Dilution of Precision (PDOP) of 3, the positioning error caused by the ionosphere can be as much as 150 meters during high solar activities period such as the current year of 2002. Another way of mitigating the ionospheric effect is to use various data processing techniques such as forming the double differences of the GPS observables. For point positioning purposes, we usually do not have the luxury of doing this. However, we can use other empirical or physics-based first principle models to mitigate about 50% RMS of the ionospheric delay (Langley, 1996).

It is also possible to use wide area differential GPS (WADGPS) corrections to mitigate error sources such as satellite clocks, satellite ephemerides and the ionospheric delay (Wells et al., 1996; Parkinson et al. 1996). To provide accurate ionospheric delay corrections for single frequency GPS users, WADGPS systems must broadcast ionospheric delay estimates derived from reference networks of dual-frequency GPS receivers. Global receiver networks have been used for many years to measure and map ionospheric total electron content (TEC) hence ionospheric delays on global scales. In particular, Global Ionospheric Mapping (GIM) software developed at the NASA Jet Propulsion Laboratory uses observations from about 100 GPS sites to compute global maps of vertical TEC with 15-minute time resolution and about 5-degree spatial resolution. The vertical variation of the ionospheric electron density is represented by a simplified, predetermined form consisting of a constant density slab at fixed height with exponential tails (Ho et al, 1996, Mannucci et al. 1998). Regional TEC maps such as those produced by Jakowski et al. (1996) and Schaer et al. (1996) could also be used to provide WADGPS-type ionospheric corrections.

In this paper we report on recent developments we made concerning a WADGPS ionospheric model. The new model is a simple extension of the currently used single-shell ionospheric model. We extended the model by including two more shells to solve for horizontal basis functions on three separate shells. We validate the new model by excluding a handful of GPS sites from the solution and then predict the slant ionospheric delay for the stations removed. As a measure of precision for prediction, we form the RMS of differences between the predicted and measured ionospheric delays.

## ESTIMATION STRATEGY

The ionospheric measurements from a GPS receiver can be modeled with the commonly used single-layer ionospheric model using the observation equation:

$$TEC = M(h, E) \sum_i C_i B_i(lat, lon) + b_r + b_s \quad \text{Eq.1}$$

where

- $TEC$  is the slant Total Electron Content using the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables where 1 TEC Unit ( $10^{16}$  electron/m<sup>2</sup>) corresponds to about 0.163 meter ionospheric delay on the L1 frequency,
- $M(h, E)$  is the thin-shell mapping function for ionospheric shell height  $h$  and satellite elevation angle  $E$ ,
- $B_i$  is the horizontal mapping function ( $C^2$ , TRIN, etc),
- $C_i$  are the basis function coefficients,
- $b_r, b_s$  are the satellite and receiver differential biases.

The new, modified model includes three distinct shells described by the following observation equation:

$$TEC = M(h_1, E) \sum_i C_{1i} B_i(lat, lon) + M(h_2, E) \sum_i C_{2i} B_i(lat, lon) + M(h_3, E) \sum_i C_{3i} B_i(lat, lon) + b_r + b_s \quad \text{Eqn.2}$$

where

- $M(h_i, E)$  is the thin shell mapping function for shell 1, etc,
- $C_{1i}$  are the basis function coefficients solved for in the filter, indexed by horizontal (i) and vertical (1,2,3 for three shells) indices.

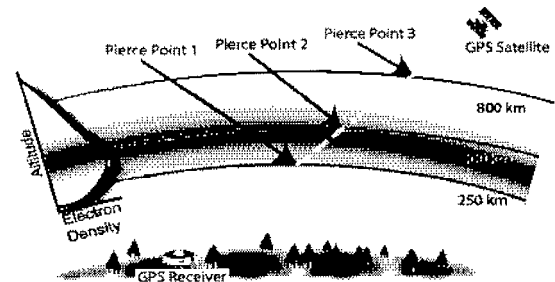


Figure 1. Illustration for the multi-shell model.

Figure 1 illustrates the three shells set at 250, 400 and 800 km. We found that using a different combination of three ionospheric shells did not reduce the ionospheric residuals any further (Komjathy et al. 2002). As shown in Figure 1, the line-of-sight vector pierces the ionosphere at three separate pierce points. The slant TEC data are converted to the vertical using the obliquity function  $M(h_p, E)$  separately computed for all three shells. The vertical TEC dependence on latitude and longitude is parameterized as linear combination of basis functions  $B_i$  with coefficients  $C_i$  as a function of solar-geomagnetic longitude and latitude. Using the phase-levelled ionospheric observable, the Kalman filter simultaneously solves for the instrumental biases and the coefficients  $C_i$ . The coefficients  $C_i$  are allowed to vary in time as a random walk stochastic process (Iijima et al. 1998). The basis functions currently used are locally supported basis functions based on a bicubic spline technique developed at JPL (Lawson, 1984).

## DATA PROCESSING

For our test data set, we chose a quiet and a storm day, 5 April and 6 April 2000 respectively, using GPS receivers at the Continuously Operating Reference Sites (CORS) network maintained by the US National Geodetic Survey (NGS) (CORS, 2002).

In Figure 2, we plotted the locations of 90 GPS reference stations. Using arrow symbols, we indicated station locations that we later removed from the solution for validation purposes. In Figure 2, the circle represent the CORS sites while the triangles correspond to the available International GPS Service (IGS) stations. It is shown that the CORS sites provide good spatial coverage while we only have only have about 17 IGS sites within the conterminous United States (CONUS) region with poor spatial distribution.

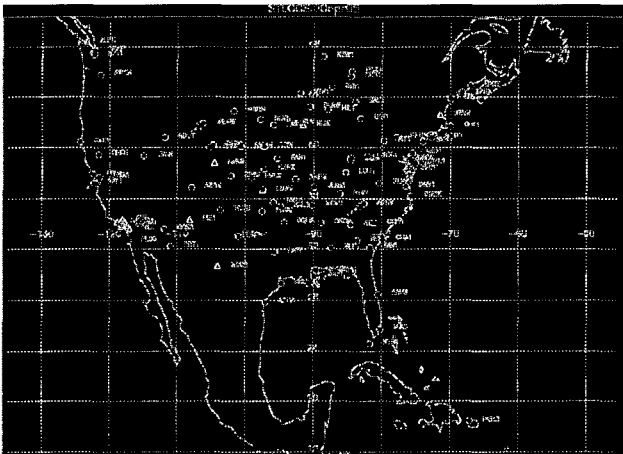


Figure 2. Network of CORS and IGS stations processed for April 5, 2000.

In the next section, we present methods to validate the new multiple shell approach: performing "missing" site tests. The missing site approach uses a network of dual-frequency GPS receivers within the CONUS but excludes a handful as validation sites. The line-of-sight TEC at these missing sites is predicted, using the GIM solutions, and then validated against the actual line-of-sight TEC observations. The inter-frequency receiver biases for the missing sites are estimated in a separate run that includes all sites. Comparisons using data, both quiet and disturbed are presented using both the single shell and the new multiple shell approaches.

## ANALYSIS OF RESULTS

In the first part of the data analysis, we looked at the estimated satellite and receiver instrumental biases using the old, single shell and the new multi-shell approaches. We compared the bias scatter (std. dev.) over a 7-day period. In Figure 3, we found that the standard deviation over the 7-day period improved by a factor of 2 to 4. In case of the satellite biases, the standard deviation improved from 2–6 cm to 8–24 mm.

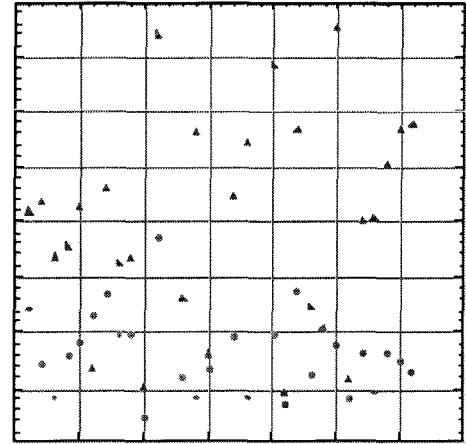


Figure 3. Comparison of satellite instrumental bias estimates.

A similar trend can be seen in the receiver bias estimates as shown in Figure 4. Using the multi-shell approach, the 7-day scatter improved from 8–64 cm to 0.5–19 cm. In Figure 4, the larger scatter values are due to stations in southern latitudes where some of the un-modeled ionospheric errors may have propagated into the bias estimates. This is due to the fact that based on Eqns 1 and 2, we estimate the sum of the line-of-sight ionospheric delays and the instrumental biases. Any systematic un-

modeled ionospheric effect will propagate into the bias estimates.

#### Receiver Bias Uncertainty: Multi-Shell versus Single-shell

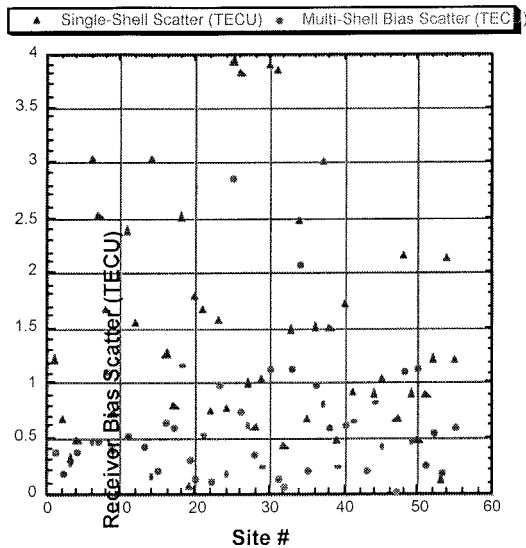


Figure 4. Receiver instrumental bias estimates.

The second part of the data analysis is concerned with the comparison of line-of-sight ionospheric residuals using single-shell and multi-shell approaches. Figure 5 shows a subset of CORS stations using both approaches. In the figure, it is shown that we achieved improvement in the ionospheric delay residuals over the single shell technique for all stations. The RMS of the ionospheric residuals is always smaller than 0.7 meter.

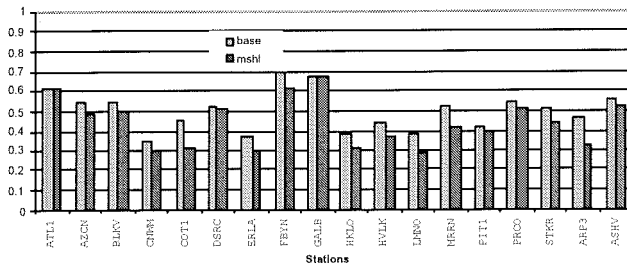


Figure 5. RMS of Postfit Residuals for CORS.

We also processed separately the available IGS stations for 5 April 2000. In Figure 6, we plotted the RMS of the postfit ionospheric delay residuals. The RMS values are similar or smaller than those using the CORS stations. However, it is interesting to see that the multi-shell improvement over single-shell is more pronounced in the case of the IGS stations, mainly due to the fact that the IGS stations are less affected by the multipath than the CORS sites.

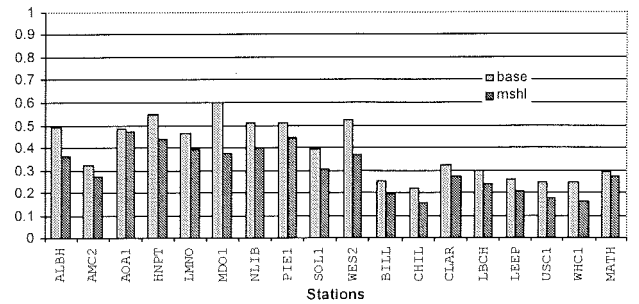


Figure 6. RMS of Postfit ionospheric residuals for IGS.

As validation for the new multi-shell approach, we removed six CORS stations from the network of 90 receivers. We selected the six sites to be evenly distributed within the CONUS (see Figure 2). We computed a solution using the remaining 84 stations and then computed the ionospheric residuals by forming the differences between the predicted line-of-sight ionospheric delay and the actual measured ones at the six test sites. In Figure 7, we plotted both the RMS of the postfit residuals and the RMS of the prediction residuals. In Figure 7, it is seen that multi-shell approach does better than the single-shell for all sites also in prediction mode. The RMS of the prediction residuals are larger by about 0.1 meter in comparison with the corresponding RMS of the postfit residuals.

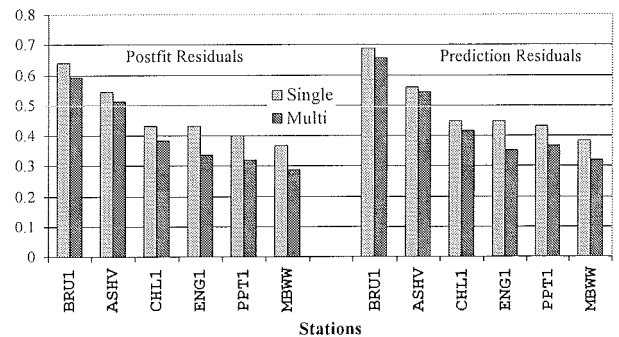


Figure 7. Comparison of Postfit and Prediction Residuals.

The data set we have discussed thus far is that of 5 April 2000 and characterized by quiet ionospheric conditions. We also processed the subsequent day during which a major geomagnetic event occurred with an  $A_p$  of 236. The RMS of postfit residuals for five stations are summarized in Figure 8 indicating both the quiet and storm day conditions. Station MBWW was unavailable for April 6, 2000. Figure 8 clearly shows that for 4 of the 5 stations investigated, the RMS of postfit residuals increased for the storm day. The RMS increased by 0.1–0.2 meter. It is evident that multi-shell performed well even for the storm day compared to the single-shell approach.

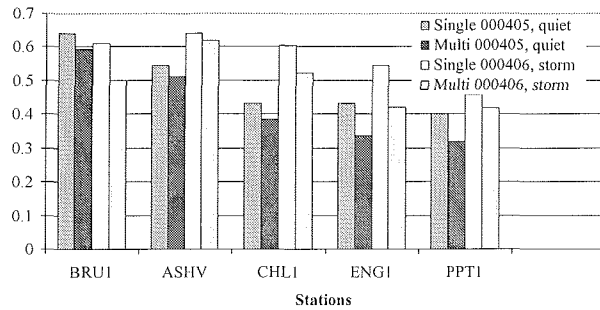


Figure 8. Comparison of postfit residuals between quiet and storm days.

As an example in Figure 9, we included a plot of the line-of-sight ionospheric delay as a function of elevation angle. The ionospheric delay ranges between 5 and 45 meters. This range of slant ionospheric delays is typical for conditions near the peak of a solar cycle such as the year 2000.

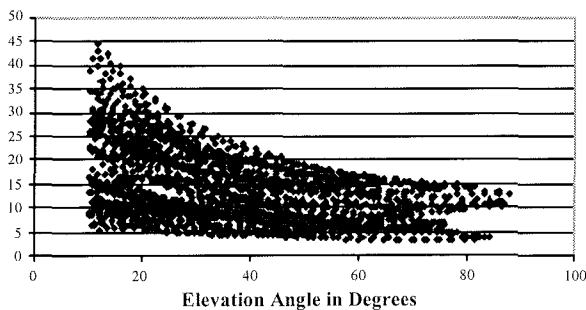


Figure 9. Slant ionospheric delay at station ENGI.

In Figure 10, we plotted the ionospheric residuals computed by differencing the predicted slant ionospheric delay and measured ones. It is shown in the figure that the multi-shell approach reduced the RMS of residuals from 0.46 to 0.37 meter. Also, it is also evident that significant improvement was achieved over the single shell approach at the low elevation angle regime.

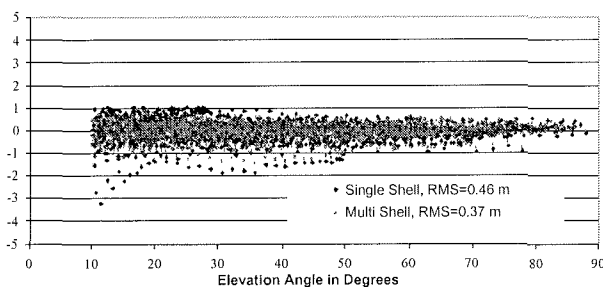


Figure 10. Ionospheric residuals for station ENGI.

Another example is shown in Figure 11 for station ASHV (Ashville, NC). It is interesting to see that the RMS of prediction residuals is reduced from 0.61 to 0.54 meter using the multi-shell approach. However, notice a bi-modal residual behaviour being more pronounced on

Figure 12. In the figure we can clearly see a systematic error showing a diurnal dependence. This is to demonstrate that there is always room for improvement in our ionosphere modeling technique by exploring parameter space further to improve predictions.

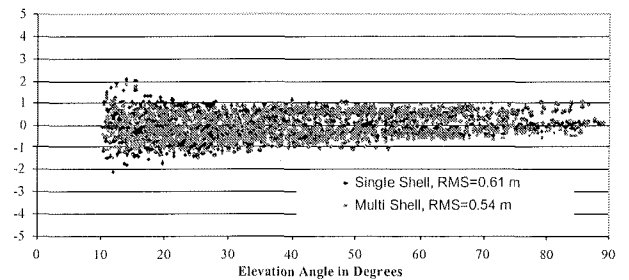


Figure 11. Ionospheric residuals for station ASHV.

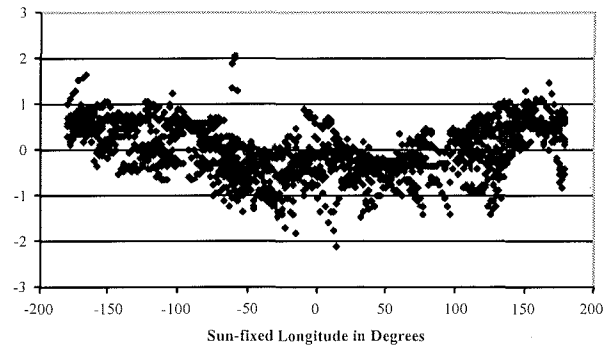


Figure 12. Ionospheric residuals as a function of Sun-fixed longitude.

In summary, we investigated both the CORS and IGS sites from the CONUS sector for both quiet and storm days. Based on this limited data set, we found that we were able to achieve a better than 0.7 meter of RMS of ionospheric residuals.

User positioning accuracy is defined as the product of Position Dilution of Precision (PDOP) and the measurement accuracy. Assuming a slant ionospheric measurement error of 0.7 meter and an average PDOP value of 3, we conclude that the user positioning error is up to 2.1 meters caused by the ionosphere.

Using the multi-shell approach over the single-shell one does not offer dramatic improvement in positioning accuracy. However, it can provide significant improvement in predicting slant ionospheric delays at low elevation angles. This in turn will provide an improved WADGPS positioning accuracy when using data from low elevation angle satellites.

## CONCLUSIONS AND FUTURE RESEARCH

In this paper, we introduced a new ionospheric model for WADGPS applications being a simple extension of the

currently available instantaneous fit models. We no longer assume that the ionosphere can be approximated with a single centroid height, but we assume three separate shells that make up the ionosphere. This new multi-shell ionospheric model for WADGPS significantly reduces day-to-day scatter of biases. It also reduces RMS of postfit residuals for all stations compared to single-shell. It is shown that multi-shell reduces residuals at low elevation angles. In the paper we demonstrated that multi-shell does better than single-shell in the mid-latitude sector in prediction mode. Being able to offer better slant TEC prediction accuracy can ensure us to achieve improved user positioning accuracy using GPS data from low elevation angle satellites.

We found that the CORS sites provided better spatial coverage over the CONUS region. On the other hand, IGS sites are less affected by multipath and so multi-shell shows better improvement over single-shell using IGS sites.

We found that the RMS of ionospheric delay residuals is better than 0.7 meter for both quiet and storm days. Assuming a PDOP of 3, this will correspond up to 2.1 meter positioning error caused by the ionosphere.

## ACKNOWLEDGMENT

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